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Motion in a Microgravity Environment

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The microgravity environment inside a space station in circular orbit around the Earth is described. The linear equations of motion of a freely falling test particle in a space station under the influence of gravity gradient and atmospheric drag is explicitly determined. These equations are solved for some simple cases, with the idealized assumption of constant drag, to produce a qualitative description of the motion of a test particle over an orbital period.

Introduction

ASSEMBLY of the International Space Station (the space station) has begun and will continue for several years, resulting in the creation of an advanced orbiting platform for scientific experiments and for observations of the Earth, the solar system, the galaxy, and the universe beyond. Scientific experiments will be performed in what is termed a microgravity environment, and this will allow for a unique set of research and development efforts that are precluded in the 1-g environment found on the surface of the Earth. As will be shown presently, the gravitational acceleration found at the Earth's surface ($1g = 9.8$ m/s^2) is diminished by six orders of magnitude in orbit, so that natural levels (that is, excluding artificial sources such as thruster firings and induced vibrations) are at the level of a micro-g, or μg, where $1\ \mu g = 10^{-6}\ g$.

Note, however, that the μg environment is not a zero-g environment because true zero-g will occur only at the center of mass of the space station and only if there is no atmospheric drag or other induced accelerations. In general, there will be a small residual acceleration field within the space station due to vibration, drag, and gravity gradient (which is the small residual force remaining when gravitational and centrifugal forces have nearly cancelled close to the center of mass). Vibration will not be considered here, but drag and gravity gradient will be. Atmospheric drag has often been considered, for example, in its effect on satellite orbits[1] and spacecraft reentry dynamics.[2] Here, we consider the effect of atmospheric drag on motion inside the space station during one orbital period ($T \cong 5400$ s). A mathematical description of atmospheric drag and gravity gradient effects on internal motion in the space station is the purpose of this paper. The development here should be of use to future space station principal investigators in understanding the space station microgravity environment.

Basic Orbital Motion

The mathematical description of the μg environment begins with Isaac Newton's laws of motion and gravity. In this section, motion of the space station as a whole is considered. Let M be the space station mass, $\boldsymbol{R}$ be its position vector with respect to the center of the Earth, $\boldsymbol{V} = \mathrm{d}\boldsymbol{R}/\mathrm{d}t$ its velocity, and μ be the gravitational parameter of the Earth ($\mu = 3.986 \times 10^{14}$ m^3/s^2 and is distinct from μg). First, assume that the Earth has spherical symmetry, so that oblateness and higher-order effects are neglected. Next, let the drag force, whose magnitude is usually written[1] as $f_D = \rho V^2 S C_D/2$ and whose direction is opposite to $\boldsymbol{V}$, be denoted by $\boldsymbol{f}_D = -\gamma M \boldsymbol{V}$, where $\gamma = \rho S C_D V/2M \geq 0$, ρ is the atmospheric density, S is the projected area in the direction of $\boldsymbol{V}$, and $C_D \cong 2.35$ is the drag coefficient (as determined from space station simulations and related in a private communication with M. R. Laible of The Boeing Company, Houston, Texas, in December 1999). Also, lift is neglected here. Lift is an oscillating quantity (due, for example, to solar array rotation) and the ratio of average lift to average drag over an orbit for the space station is estimated to be somewhere between 5 and 20% (as related in two private communications, one with C. D. Chlouber of The Boeing Company, Houston, Texas, in December 1999 and the other with F. E. Lumpkin of NASA Johnson Space Center, Houston, Texas, also in December 1999). Here, for the purpose at hand, the drag force $\boldsymbol{f}_D$ will be taken as small and constant (to first order) for a single orbit and lift will be set to zero.

The motion of the space station center of mass is then described by the following differential equation:

$$\frac{\mathrm{d}M\boldsymbol{V}}{\mathrm{d}t} = -M\frac{\mu}{R^3}\boldsymbol{R} - \gamma M\boldsymbol{V} \tag{1}$$

If the velocity $\boldsymbol{V}$ is dotted into Eq. (1), the result is

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -\gamma V^2, \qquad \text{where} \qquad E = \frac{1}{2}V^2 - \frac{\mu}{R} \tag{2}$$

The quantity E (the specific energy) clearly decreases with time if $\gamma > 0$. The speed V, in terms of the orbital period Ω and distance R, is $V = R\Omega$, to first order in γ. Here it is assumed that the decay of the space station circular orbit is slow, so that $\gamma \ll \Omega$ and the balance between centrifugal and gravitational forces will hold at any time. Then, from Eq. (1), this balance yields $R^3\Omega^2 = \mu$, to second order in γ. Putting this into Eq. (2) leads to

$$E = -\frac{1}{2}\frac{\mu}{R}, \qquad \dot{R} = \frac{\mathrm{d}R}{\mathrm{d}t} = -2\gamma R < 0 \tag{3}$$

Thus, R decreases due to atmospheric drag, as expected. Furthermore, if we differentiate the relationship $R^3\Omega^2 = \mu$, we find that $2\dot{\Omega}R = -3\dot{R}\Omega$, that is, $\dot{\Omega} > 0$. Because the orbital speed of the space station is $V = R\Omega$, the change in V is $\dot{V} = \dot{R}\Omega + R\dot{\Omega} = R\dot{\Omega}/3$ or $\dot{V} > 0$. This straightforward result, that the presence of a small atmospheric drag increases orbital velocity, is sometimes called the *drag paradox*.[3]

Linear Approximation

The space station will orbit at an altitude of between 350 and 460 km, the exact altitude depending on atmospheric density. After being set at some nominal value, altitude will decrease due to atmospheric drag and require reboost to a new operational altitude. The orbital decay will be gradual so that at any instant the orbit can be considered circular. The space station local-vertical/local-horizontal (LVLH) axes are fixed with respect to a circular orbit, with the origin of the coordinate system at the space station center of mass; the z axis (z for zenith) points radially toward the Earth's center, the x axis points along the orbital velocity vector, and the y axis completes the right-handed triad, that is, points to starboard. The orbital angular frequency, $\Omega > 0$, has an associated vector $\boldsymbol{\Omega} = -\Omega\hat{\boldsymbol{y}}$, where $\hat{\boldsymbol{y}}$ is a unit vector in the y direction (and $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{z}}$ are unit vectors for the x and z directions, respectively). If the vector from the center of the Earth to the space station center of mass is $\boldsymbol{R} = R\hat{\boldsymbol{r}}$ (where $\hat{\boldsymbol{r}}$ is a unit

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vector radially outward from the Earth's center), then a vector from the Earth's center to any point on the space station is $\boldsymbol{r} = \boldsymbol{R} + \boldsymbol{\rho}$, where, in terms of the LVLH coordinate system, the position vector is $\boldsymbol{\rho} = x\hat{\boldsymbol{x}} + y\hat{\boldsymbol{y}} + z\hat{\boldsymbol{z}}$.

The equation of motion of a test particle inside the space station will be similar to Eq. (1), except that there will be no atmospheric drag term because the station shields its own interior, although there may be other accelerations $\boldsymbol{a}$ (fluid mechanical, electromagnetic, viscoelastic, etc.). In inertial space, with the origin of the reference frame being at the Earth's center and including the possibility of additional accelerations, the equation of motion is

$$\frac{\mathrm{d}^2\boldsymbol{r}}{\mathrm{d}t^2} = -\frac{\mu}{r^3}\boldsymbol{r} + \boldsymbol{a} \tag{4}$$

The relative motion (in an inertial frame of reference) between the test particle and the space station center of mass is found by dividing out M from Eq. (1) and subtracting Eq. (1) from Eq. (4):

$$\frac{\mathrm{d}^2\boldsymbol{\rho}}{\mathrm{d}t^2} = -\left(\frac{\mu}{r^3}\boldsymbol{r} - \frac{\mu}{R^3}\boldsymbol{R}\right) + \boldsymbol{a} + \gamma\boldsymbol{V} \tag{5}$$

Because $\boldsymbol{r} = \boldsymbol{R} + \boldsymbol{\rho}$ and $R^3\Omega^2 = \mu$, we can expand Eq. (5) to first order using the binomial theorem; the result is

$$\frac{\mathrm{d}^2\boldsymbol{\rho}}{\mathrm{d}t^2} = -\Omega^2(\boldsymbol{\rho} - 3\boldsymbol{\rho}\cdot\hat{\boldsymbol{r}}\hat{\boldsymbol{r}}) + \boldsymbol{a} + \gamma\boldsymbol{V} \tag{6}$$

The space station LVLH coordinate system, described earlier, is a rotating frame of reference. The equation of motion (6), in such a rotating reference frame[4] has the following form, to first order, in γ:

$$\frac{\mathrm{d}^2\boldsymbol{\rho}}{\mathrm{d}t^2} = -\Omega^2(\boldsymbol{\rho} - 3\boldsymbol{\rho}\cdot\hat{\boldsymbol{r}}\hat{\boldsymbol{r}}) + \boldsymbol{\Omega}\times(\boldsymbol{\rho} + \boldsymbol{\Omega}) + 2\boldsymbol{v}\times\boldsymbol{\Omega} + \boldsymbol{a} + \gamma\boldsymbol{V} \tag{7}$$

Here, the velocity is $\boldsymbol{v} = \dot{\boldsymbol{\rho}} \equiv \mathrm{d}\boldsymbol{\rho}/\mathrm{d}t$. The first term on the right is the gravity gradient, the second term is the centrifugal acceleration, the third is the Coriolus acceleration, the fourth is any additional acceleration, and the fifth is due to atmospheric drag and can be written $\gamma\boldsymbol{V} = \beta\hat{\boldsymbol{x}}$, where $0 \le \beta \ll \Omega^2 R$. (A further rotational term $\boldsymbol{\rho}\times\boldsymbol{\Omega}$ is omitted because it is of second order in size.)

In the LVLH system $\boldsymbol{\Omega} = -\Omega\hat{\boldsymbol{y}}$, and the second and third terms on the right-hand side of Eq. (7) are, respectively,

$$\boldsymbol{\Omega}\times(\boldsymbol{\rho}\times\boldsymbol{\Omega}) = \Omega^2(\boldsymbol{\rho} - y\hat{\boldsymbol{y}}), \qquad 2\boldsymbol{v}\times\boldsymbol{\Omega} = 2\Omega(\dot{z}\hat{\boldsymbol{x}} - \dot{x}\hat{\boldsymbol{z}})$$

Putting these into Eq. (7) yields the components equations

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = 2\Omega\dot{z} + \beta + a_x \tag{8}$$

$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} = -\Omega^2 y + a_y \tag{9}$$

$$\frac{\mathrm{d}^2 z}{\mathrm{d}t^2} = 3\Omega^2 z - 2\Omega\dot{x} + a_z \tag{10}$$

Equations (8–10) describe the acceleration of a small, free-falling test particle located on the space station; they can be termed the *microgravity equations of motion.* In Eqs. (8–10), fluid dynamic, electromagnetic, and viscoelastic accelerations are omitted, but could be introduced as necessary through $\boldsymbol{a}$. Note also that if $\beta > 0$ (i.e., if there is any drag at all), then zero-g is not possible for experiments at fixed positions within the space station. However, any small object that is moving freely within the space station will experience practically zero-g (apart from forces due to air currents), until a restraining structure, such as a wall, is reached. As will be seen shortly, the time it takes to reach a wall is generally only a small fraction of an orbital period.

The linearization of the gravitational force equations (without drag) to determine the relative motion of objects in nearby orbits has been previously accomplished and solutions to the drag-free equations have been found. Among those who have worked on this problem are Hill,[5] Wheelon,[6] and Clohessy and Wiltshire.[7] There are a number of modern texts which describe this work,[2,8–10] and linearized solutions are still very much a part of current research.[11] In this paper, the previous work is extended by including small orbital drag forces in the equations.

Solution of the Microgravity Equations of Motion

Consider now Eqs. (8–10) with $\boldsymbol{a} = \boldsymbol{0}$. Equation (9), for the y coordinate, is uncoupled from the other two and, with no additional forces, has the simple solution

$$y = y_0\cos\Omega t + (\dot{y}_0/\Omega)\sin\Omega t \tag{11}$$

Equations (8) and (10) are coupled together and depend on the exact nature of the atmospheric drag, which manifests itself in the time dependence of β. In general, a numerical solution is required. However, if β is treated as a small constant for didactic purposes, then Eq. (8) can be immediately integrated to yield

$$\dot{x} = \dot{x}_0 + 2\Omega(z - z_0) + \beta t \tag{12}$$

Placing Eq. (12) into Eq. (10) gives

$$\frac{\mathrm{d}^2 z}{\mathrm{d}t^2} = -\Omega^2 z - 2\Omega(\dot{x}_0 - 2\Omega z_0 + \beta t) \tag{13}$$

The general solution to Eq. (13) is

$$z = z_0 + (\dot{z}_0/\Omega)\sin\Omega t + (3z_0 - 2\dot{x}_0/\Omega)(1 - \cos\Omega t) - (2\beta/\Omega^2)(\Omega t - \sin\Omega t) \tag{14}$$

Putting this into Eq. (12) and integrating produces

$$x = x_0 + \dot{x}_0 t - \tfrac{3}{2}\beta t^2 + 2\left(\dot{z}_0/\Omega + 2\beta/\Omega^2\right)(1 - \cos\Omega t) + 2(3z_0 - 2\dot{x}_0/\Omega)(\Omega t - \sin\Omega t) \tag{15}$$

Equations (11), (14), and (15) represent the general solutions to the microgravity equations of motion (8–10), for the case when β may be approximated as a small constant (to achieve a qualitative analytic solution). If Eq. (15) is expanded out in powers of Ωt, we have

$$x = x_0 + (\dot{x}_0/\Omega)(\Omega t) + (\beta/2\Omega^2)(\Omega t)^2 + (z_0 - 2\dot{x}_0/3\Omega)(\Omega t)^3 + \cdots \tag{16}$$

Expanding Eq. (14) in a similar manner yields

$$z = z_0 + (\dot{z}_0/\Omega)(\Omega t) + (3z_0/2 - \dot{x}_0/\Omega)(\Omega t)^2 - (\beta/3\Omega^2)(\Omega t)^3 + \cdots \tag{17}$$

Here we see that the effects of drag show up first at second order in Ωt for motion in the x direction and third order for motion in the z direction. Thus, when motion is integrated for short times, drag first affects motion in the x direction, but is still negligible in the z direction. During this short time, motion of the station can be thought of as approximately tangential and atmospheric drag does appear to deaccelerate the vehicle. However, for longer times, Eq. (15) shows that the space station speed actually increases due to drag because the trajectory is really a curved one.

The solutions (11), (14), and (15) are valid as long as $\rho \ll R$, $\dot{\rho} \ll V = R\Omega$, and $\beta \ll \Omega^2 R$, and only over one orbit, in which case β can be taken as a (small) constant. If we set $\beta = x = y = z = 0$ in Eqs. (15), (19), and (20) and solve for $\dot{x}_0$, $\dot{y}_0$, and $\dot{z}_0$, then we have the Clohessy–Wiltshire equations,[7] which are used in iterative rendezvous guidance maneuvers. At any time, a firing of one spacecraft's thrusters to achieve $\dot{x}_0$, $\dot{y}_0$, and $\dot{z}_0$ will put that spacecraft on an approximate trajectory to rendezvous with another spacecraft (located at the origin of the LVLH coordinate system).

Again, note that with $\beta \neq 0$, Eqs. (11), (14), and (15) describe motion in an LVLH frame of reference that is itself subjected to atmospheric drag. Thus, the solutions pertain to the motion of objects inside of the space station, where orbital drag does not affect motion directly. Instead, drag affects the station, and the associated acceleration of the station causes apparent forces to appear on objects inside the station (which shields these objects from any direct effect of orbital drag). The relative motion between two different spacecraft, each of different mass, ballistic coefficient and subject to differing amounts of atmospheric drag (e.g., due to shadowing effects and to varying ballistic coefficient) is another problem altogether. However, if β is the relative drag between the two different spacecraft, then it may be possible to use Eqs. (11), (14), and (15) to more precisely define rendezvous and docking maneuvers.

Discussion

Consider Eqs. (8–10). If an object is fixed with respect to the LVLH frame and undergoes no acceleration with respect to that frame, then structural forces must produce apparent accelerations $\boldsymbol{a}$ that balance the drag and gravity gradient terms that appear on the right-hand sides. In this case, the components of the apparent acceleration field satisfy $a_x = -\beta$, $a_y = \Omega^2 y$, and $a_z = -3\Omega^2 z$. The first is due to drag, and a typical value[12] of this acceleration is $\beta = 0.2\ \mu g$. Also, if $\Omega^2 = (2\pi/T)^2 = 1.35 \times 10^{-6}\ \text{s}^{-2}$, then $a_y = 1\ \mu g$ when $y = 7.2$ m and $a_z = 1\ \mu g$ when $z = 2.4$ m. Thus, in the y–z plane, contours of constant $(a_y^2 + a_z^2)^{1/2}$ are ellipses, where the 1-μg ellipse has a semimajor axis of 7.2 m and a semiminor axis of 2.4 m (higher value μg ellipses scale linearly with these values). These ellipses, centered on the station center of mass, are shown in Fig. 1, where a head-on, representative (not necessarily accurate) view of the space station is given for illustrative purposes. Because the laboratory modules in Fig. 1 all have quasi-steady acceleration fields of only a few μg, they are generally said to have a microgravity environment. Note that there is no point of zero-g because $a_x = -\beta$, and $\beta > 0$, in general.

Now, consider the motion of a test object released from rest, over one orbital period. For comparison, use $\beta = 0$ and $0.2\ \mu g$, and at $t = 0$, set $\rho_0 = 0$ and $v_0 = (0, 0, 0.1)$ m/s. The results, for $\beta = 0$ and $0.2\ \mu g$ are shown in Figs. 2 and 3, respectively. There are two important points to note. First, the range of motion in both cases is large and would require an enormous interior to the station so that the object maintains freedom of motion. Second, when $\beta = 0$, the object returns to its initial location, and when $\beta = 0.2\ \mu g$ the object winds up above and behind its initial location (because the station is speeding up and losing altitude). [Many more interesting figures can be generated using Eqs. (14) and (15), but this will not be done here.]

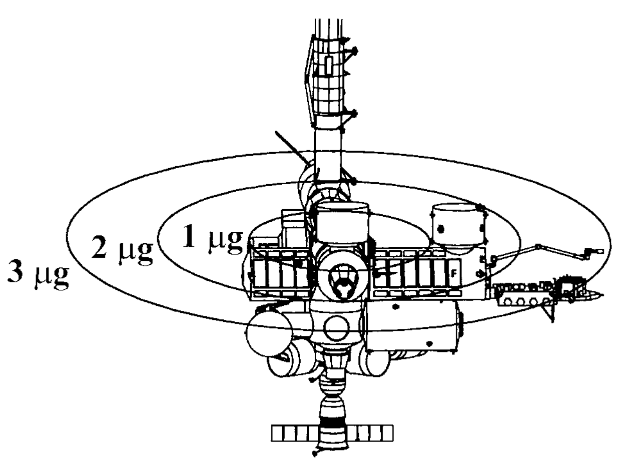

Fig. 1 Head-on representation of the space station, showing microgravity contours.

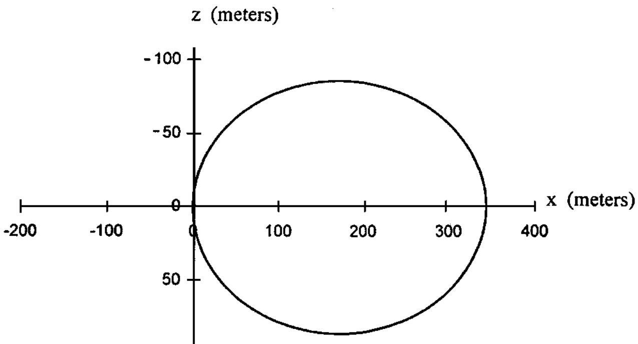

Fig. 2 Motion of a test particle for one orbit, with no drag on the space station.

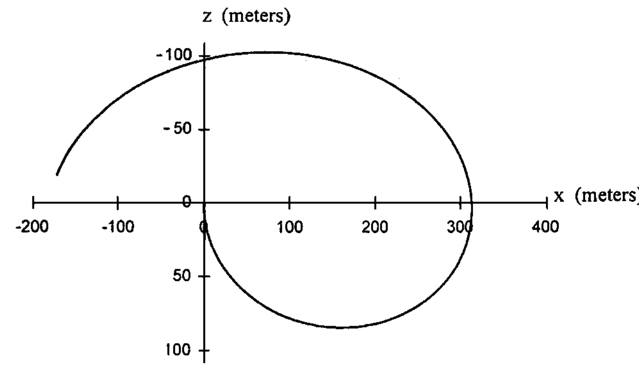

Fig. 3 Motion of a test particle for one orbit, with a 0.2-θg drag on the space station.

Conclusion

The linearized equations of motion for an object inside an orbiting space station have been derived and solved. Over an orbit, the time available for unrestricted motion is generally small compared to an orbital period, so that complete weightlessness is not usually possible. Even close to the center of mass of the space station (somewhere within node 1 at assembly complete), the presence of orbital drag causes freely floating objects to move into contact with restraining structure within a few minutes. For example, assuming that the average drag is $0.2\ \mu g$, an object placed motionless at the space station's center of mass will appear to move about 1 m in one-fifth of an orbit. The quasi-steady forces that are encountered anywhere in the space station, however, are only in the range of a few μg, a small amount that is usually negligible, except for highly sensitive on-orbit experiments.

It is hoped that the development presented in this paper will provide space station principal investigators and other interested parties detailed answers to those questions that naturally arise when they first begin to seriously consider how motion in a microgravity environment can be accurately described. The sources of μg acceleration considered here are gravity gradient and atmospheric drag; higher frequency vibrations, such as those resulting from normal space station operation and crew activity are also important but were not addressed here.

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